RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. FIRST SEMESTER EXAMINATION, DECEMBER 2019
FIRST YEAR [BATCH 2019-22]
MATHEMATICS (Honours)
Paper : II [CC2]

Date : 13/12/2019
Time : 11 am – 1 pm
Full Marks : 50

[Use a separate Answer Book for each group]

Group – A

Answer **any three** questions from Question Nos. 1 to 5 : [3×5]

1. Show that the equation of the line joining the feet of perpendiculars from the point (d,0) on the lines $ax^2+2hxy+by^2=0$ is $(a-b)x+2hy+bd=0$.

2. Prove that the two conics $\frac{l_1}{r}=1-e_1\cos\theta$ and $\frac{l_2}{r}=1-e_2\cos(\theta-\alpha)$ will touch one another, if $l_1^2\left(1-e_2^2\right)+l_2^2\left(1-e_1^2\right)=2l_1l_2\left(1-e_1e_2\cos\alpha\right)$.

3. If P,Q are two points on the conic $\frac{l}{r}=1-e\cos\theta$ with $\alpha-\beta$ and $\alpha+\beta$ as the vectorial angles, where β is a constant, show that the locus of the foot of perpendicular from the pole on the line PQ is $\left(e^2-\sec^2\beta\right)r^2+2ler\cos\theta+l^2=0$.

4. Reduce the equation $11x^2-4xy+14y^2-58x-44y+71=0$, to its normal form. State the nature of the conic and find its eccentricity. [4+1]

5. The normal at a variable point P on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ cuts the diameter CD, conjugate to CP, at Q. Show that the locus of Q is $\frac{a^2}{x^2}+\frac{b^2}{y^2}=\left(\frac{a^2-b^2}{x^2+y^2}\right)^2$, C being the centre of the ellipse.

Answer **any two** questions from Question Nos. 6 to 8 : [2×5]

6. Show that the vectors $\hat{i},\hat{i}+\hat{j},\hat{i}+\hat{j}+\hat{k}$ are non-coplanar. Express the vector $3\hat{i}+4\hat{j}+5\hat{k}$ in a linear combination of those vectors. [2+3]

7. a) Solve $\vec{a}\times\vec{x}=\vec{b}$ and $\vec{a}.\vec{x}=c$, where $\vec{a}$ and $\vec{b}$ are any two vectors and c is a scalar.
   b) Find the torque about the point $\left(2\hat{i}+\hat{j}-3\hat{k}\right)$ of a force represented by $\left(\hat{i}+2\hat{j}+\hat{k}\right)$ passing through the point $\left(3\hat{i}+4\hat{j}-\hat{k}\right)$. [3+2]

8. Show that $\left(\vec{b}\times\vec{c}\right)\cdot\left(\vec{a}\times\vec{d}\right)+\left(\vec{c}\times\vec{a}\right)\cdot\left(\vec{b}\times\vec{d}\right)+\left(\vec{a}\times\vec{b}\right)\cdot\left(\vec{c}\times\vec{d}\right)=0$. Use it to show that $\sin(A+B)\sin(A-B)=\sin^2 A-\sin^2 B$ for any two acute angles A and B. [2+3]

Group – B

Answer **any five** questions from Question Nos. 9 to 16 : [5×5]

9. Show that the differential equation $\left(x^2-4xy-2y^2\right)dx+\left(y^2-4xy-2x^2\right)dy=0$ is exact, and find its general solution.

10. Solve the differential equation $x\dfrac{dy}{dx}=y+\sqrt{x^2+y^2}$.

11. Write down the differential equation $\left(y-1\right)\dfrac{dy}{dx}-x.\left(\dfrac{dy}{dx}\right)^2+2=0$ in its Clairaut's form. Then find its general and singular solution. [1+4]

12. Solve, by the method of variation of parameters, the following differential equation:
$$\frac{d^2y}{dx^2}+y=\text{cosec } x.$$

13. Solve the differential equation
$$\frac{d^2y}{dx^2}-4\frac{dy}{dx}+4y=x^3e^{2x}+xe^{2x}$$ by the method of undetermined coefficients.

14. Find the general solution of the differential equation $\left(x+1\right)^2\dfrac{d^2y}{dx^2}-2\left(x+1\right)\dfrac{dy}{dx}+2y=1$ given that $y=x+1$ and $y=\left(x+1\right)^2$ are linearly independent solutions of the corresponding homogeneous equation.

15. Solve the differential equation $2y\dfrac{d^3y}{dx^3}+6\dfrac{dy}{dx}.\dfrac{d^2y}{dx^2}=-\dfrac{1}{x^2}$

16. Find the equation of a set of curves, each member of which cuts every member of the family xy = constant at the angle $\dfrac{\pi}{4}$

——— × ———